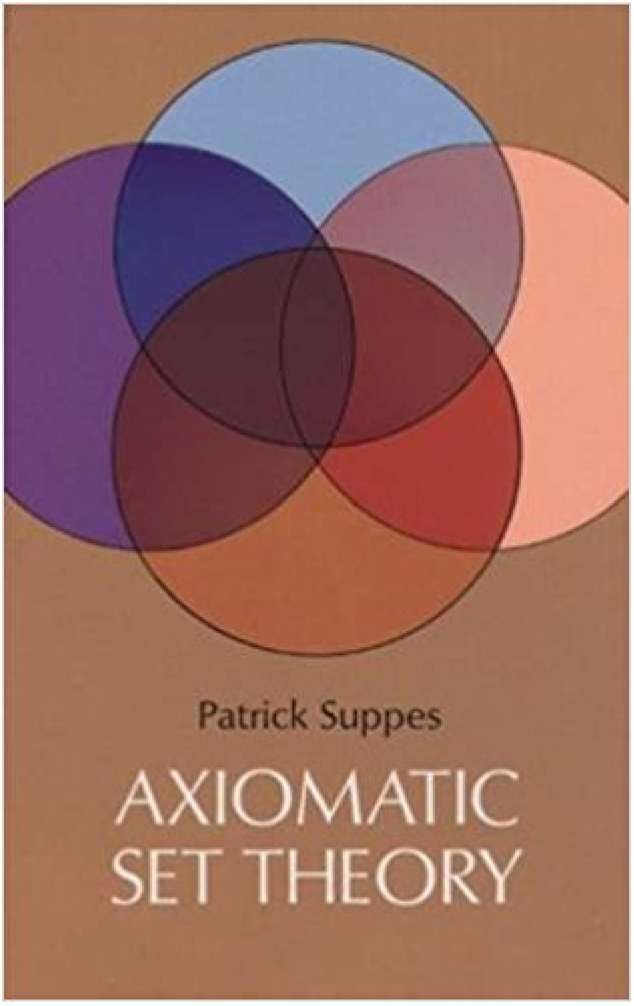
# Axiomatic Set Theory (Dover Books On Mathematics)



## Synopsis

One of the most pressingproblems of mathematics over the last hundred years has been the question: What is a number? One of the most impressive answers has been the axiomatic development of set theory. The question raised is: "Exactly what assumptions, beyond those of elementary logic, are required as a basis for modern mathematics?" Answering this question by means of the Zermelo-Fraenkel system, Professor Suppes' coverage is the best treatment of axiomatic set theory for the mathematics student on the upper undergraduate or graduate level.The opening chapter covers the basic paradoxes and the history of set theory and provides a motivation for the study. The second and third chapters cover the basic definitions and axioms and the theory of relations and functions. Beginning with the fourth chapter, equipollence, finite sets and cardinal numbers are dealt with. Chapter five continues the development with finite ordinals and denumerable sets. Chapter six, on rational numbers and real numbers, has been arranged so that it can be omitted without loss of continuity. In chapter seven, transfinite induction and ordinal arithmetic are introduced and the system of axioms is revised. The final chapter deals with the axiom of choice. Throughout, emphasis is on axioms and theorems; proofs are informal. Exercises supplement the text. Much coverage is given to intuitive ideas as well as to comparative development of other systems of set theory. Although a degree of mathematical sophistication is necessary, especially for the final two chapters, no previous work in mathematical logic or set theory is required.For the student of mathematics, set theory is necessary for the proper understanding of the foundations of mathematics. Professor Suppes in Axiomatic Set Theory provides a very clear and well-developed approach. For those with more than a classroom interest in set theory, the historical references and the coverage of the rationale behind the axioms will provide a strong background to the major developments in the field. 1960 edition.

## Book Information

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## Customer Reviews

This book is a basic reading on the Set Theory field. It works as an introductory and reference text, with a clear and concise style and a precise logic sequence on its exposition and development of contents. After reading it, I started to focus on more specific topics as Axiom of Choice and Banach-Tarski paradox, and I felt that it has prepared me to go further in my Math studies.

This is an excellent book for an undergraduate who has to begin doing abstract proof and has not had even rudimentary logic and symbolism.

Although this book was published 45 years ago, scientific discoveries since have been vindicating Reginald Kapp's continuous creation hypothesis, (which he first published in 1940) and are casting increasing doubt on the validity of the rival big bang theory. Some examples are: Quantum physics shows particle/wave duality, non-locality, entanglement (Goswami) Cell biology shows that genes and DNA do not control our biology, but can be re-written by our learned behaviour (Lipton) Molecules are being created all the time. Morphic fields (Sheldrake) Nobody seems to have any better explanation for gravity, which Kapp postulated is a wave caused by the disappearance of matter and its associated space. The appendices give a rich field of potential research projects.

An excellent followup to Halmos' "Naive Set Theory". For example, it proves that mathematical induction is a valid schema. It uses the Zermel-Fraenkel approach that everyone should be familiar with. As one reviewer pointed out, you're not done when you finish this book. You should proceed to J Donald Monk's "Introduction to Set Theory" where you will learn about the NBG axioms, classes and large cardinals. Suppes actually points you to NBG in his comments. Perhaps the best thing about this book is its mixture of rigor and commentary. I actually had fun reading it. Few things are better than having fun and learning math at the same time.

There is a common pedagogical attitude that studying the foundations of mathematics is a waste of time--basically, that one learns the rules of mathematical proof simply by lots of hands-on practice. This attitude is understandable, but I believe most serious math students, at some point in their training, will benefit from spending some time down at the foundations. And, with set theory being the language of rigorous mathematics, it leads straight to this book. In a nutshell, most mathematical objects (e.g. numbers and functions) can be formally defined as a set, and the axioms lay out precisely the acceptable ways of creating new sets from existing ones.As Halmos writes in "Naive Set Theory", the working mathematician learns set theory so that they can forget it. Phrased differently, it is a mathematician's chance to eliminate any gaps in their understanding of the fundamentals before moving into an active research field. Giving this objective, it is critical that a book on set theory emphasize precision and rigor; the logical structure of each proof should be as transparant, even mechanical, as possible. Suppes' book, with its heavy reliance on the symbolism of predicate calculus, fits this bill beautifully. (Ironically enough, "Naive Set Theory" did not--it was too brief and too informal for my taste.)For the record, chapters 2 and 3 are reasonably straightforward, there is a steady increase in mathematical sophistication with each chapter after that. There are abundant meaningful exercises, and I feel the text itself prepares you well to tackle them. Also, if you find the introduction (chapter 1) too difficult, you can easily jump straight to chapter 2.

Mathematics is a first order theory whose primitive formulaeall take the form 'a is a member of b'. 'a' can be a set or atom; 'b' must be a set. If you do not object to the preceding sentence, then read on.Axiomatic Set Theory (AST) lays down the axioms of the now-canonical set theory due to Zermelo, Fraenkel (and Skolem), called ZFC. Building on ZFC, Suppes then derives the theory of cardinal and ordinal numbers, the integers, rationals, and reals, and the transfinite--Cantor's paradise. Suppes accomplishes in 250 well laid out pages what required 800 crabbed pages in Principia Mathematica.This book evolved out of a class Suppes taught at Stanford in the long ago 1950s. It has since remained the best book of its kind. The reason is that subsequent presentations of set theory are too difficult, too contrived, too clever by half. They disdain the basics as old hat.AST has several valuable pedagogical features.1. The introduction to relations and functions is the best I know of. I am disappointed at how little attention has been devoted to relations and relational algebra in recent decades.2. Suppes has a nice way of introducing a simple axiom, then showing that that axiom is a theorem when a more complicated axiom is later introduced. In particular, he develops the theory of cardinals by means of a temporary axiom to the effect that equipollent sets have identical cardinalities. This axiom becomes a theorem when the axiom of

Choice is introduced in the final chapter.The axiom schema of Replacement is introduced as late as possible, to enable transfinite arithmetic. He then turns around and shows that Replacement makes Subsets and Pairing redundant.In my opinion, the greatest flaw of ZFC is that defining a cardinal number requires either the axiom of Choice, or Infinity plus the subtle notion of set rank. Frege and Russell had an appealing definition: a cardinal number is an equivalence class of sets under equipollence. That definition does not work in ZFC. It does work in Quinian set theory. Suppes does a yeoman's job of battling this flaw.3. Suppes defines a finite set in the interesting way Tarski proposed in 1924.AST contains hundreds and hundreds of theorems, man of them useful classics. In many cases, the proof is an exercise. Suppes's proof are of the informal sort typical of mathematics. What AST does can be done more rigorously: type 'Metamath' into Google and see for yourself.Even though Suppes is a philosopher, this book is almost entirely a mathematical exercise. The reader will not get a good feel for how set theory is part of analytic philosophy, and how it has been a contentious subject. The writings of Fraenkel and Bar Hillel are better in these respects. Suppes does highlight the reservations re the axiom of Choice, but Cohen's proof that Choice is independent of ZF has largely laid those reservations to rest, except for those of us with constructive sympathies. AST gives no hint that Replacement and Power Set give us far more set theory than is needed in practice. Thanks to the work of Aczel and Barwise, published around 1990, we have a better idea of what it means to dispense with Regularity. Shortly after the 1960 publication of AST, Lawvere and others began to lay down the category theoretic foundation of mathematics knowns as topos theory. That theory puts ZFC in a new light. Personally, I am astonished that an axiomatization of finite sets simpler than ZF has yet to emerge.

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